Available online at www.sciencedirect.com
ScienceDirect
International Journal of Heat and Mass Transfer 51 (2008) 506–516
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

A one-dimensional transient analytical model for earth-to-air heat exchangers, taking into account condensation phenomena and thermal perturbation from the upper free surface as well as around the buried pipes

M. Cucumo [a,*], S. Cucumo [a], L. Montoro [b], A. Vulcano [a,*]

[a] Mechanical Department, University of Calabria, 87036 Arcavacata di Rende (CS), Italy
[b] Mathematics Department, University of Calabria, 87036 Arcavacata di Rende (CS), Italy

Received 17 July 2006
Available online 9 July 2007

Abstract

A one-dimensional transient analytical model is proposed to estimate the performance of earth-to-air heat exchangers, installed at different depths, used for building cooling/heating. Two independent space coordinates are considered, one in the longitudinal direction of the buried pipe and the other through the soil, in the vertical direction. With appropriate simplifications, analytical treatment is proposed to predict the temperature fields of the fluid in the pipe and of the soil in the proximity of the buried pipe, taking into account thermal perturbation of the upper free surface and the possible phase change (condensation) in the buried pipes. Moreover, the agreement with some experimental data available in the literature is very satisfactory.

Keywords: Earth-to-air heat exchangers; Fourier equation; Green's functions; Sensible heat; Latent heat; Humidity ratio

1. Introduction

In the ambit of bioclimatic architecture, earth-to air heat exchangers (EAHEs), used to cool primary air for buildings during hot weather, have been assigned an increasingly important role [1], also for the possibility of pre-heating during winter. Their design requires the accurate determination of ground temperature, a cold source to which the air gives up heat through convection. With regard to this the evaluation of the performance of EAHEs in relation to the depth at which they are installed is of fundamental importance. In general, economic considerations would lead to place the system reasonably close to the surface (at a depth of between approximately 2–3 m) for reducing installation costs, although this would imply a reduction in their effectiveness on account of the higher ground temperature at that depth. To find the right compromise between efficiency and cost, it is necessary to assess the effect of the thermal wave coming from the surface of the ground as well as the effect of the heating of the ground near the buried pipe from the air passing through the said pipe.

The proposed model has enabled us to take these different boundary conditions into account and combine them on the basis of the principle of the superposition of causes and effects, which will be shown to be applicable to the matter in question. A one-dimensional analytical solution is proposed to determine the profile of the ground temperature under the influence of EAHEs and the thermal wave from the surface. This analytical solution has been achieved through two methods: the first using Green's functions [2] and the second through a simplified analysis

* Corresponding authors. Tel.: +39 0984 494616; fax: +39 0984 494673.
E-mail addresses: m.cucumo@unical.it (M. Cucumo), alvulcano@unical.it (A. Vulcano).

0017-9310/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2007.05.006

Nomenclature

b	earth-to-air heat exchangers installation depth (m)
c_p	specific heat at constant pressure (J/kg K)
d_h	hydraulic diameter of the buried pipe (m)
erf	error function
erfc	complementary error function
h	specific enthalpy of humid air (J/kg)
h_c	heat transfer coefficient (W/m^2 K)
h_m	mass transfer coefficient (kg/m^2 s)
k	thermal conductivity (W/m K)
L	length of buried pipe (m)
$\dot{m}$	mass flow rate (kg/s)
NTU	number of transfer units
P	perimeter (m)
$\dot{Q}$	rate of heat transfer (W)
T	temperature (°C)
$\overline{T}$	temperature averaged in the period of operation (°C)
$T_{s,y}$	yearly mean surface temperature (°C)
$T_{s,d}$	daily mean surface temperature (°C)
$T_{a,d}$	daily mean air temperature (°C)
t	time, measured from the start of the day (s)
t_1	time at start of the system (s)
t_2	time at switch off of the system (s)
t^*	time at highest daily temperature (s)
t'	integration variable of t (s)
t_0	period of temperature fluctuations (s)
u	Green's function (m^{-1})
W	humidity ratio (kg$_v$/kg$_a$)
x	depth in ground (m)
x'	integration variable of x (m)
z	distance along the length of the buried pipe (m)

Greek symbols

α	thermal diffusivity (m^2/s)
Δz	distance along the length of the elementary pipe (m)
ΔT	magnitude of temperature fluctuations: $(T_{max} - T_{min})/2$ (°C)
τ	time, measured from the start of the year (s)
τ_0	time, measured from the start of the year, at highest surface temperature (s)
ω	pulsation: $2\pi/t$ (s^{-1})

Subscripts

a	air
d	daily value
G	ground
HE	heat exchanger
in	input
ℓ	liquid
m	mean (yearly/daily) value
out	output
R	saturation
s	surface
v	vapor
y	yearly value
w	wall

based on the principle of the superposition of causes and effects. The results of both approaches are in agreement with each other.

Once the wall temperature of the buried pipe has been determined, the length and the thermo-hygrometric performances can be evaluated through the use of the simplified analytical solutions obtained by Cucumo et al. [3,4]. The evaluation of the humidity ratio of the air within the buried pipes allows us to take into account the possibility of condensation in consequence of the air layer near the pipe wall reaching saturation conditions, in accordance with the approach of the heat and mass transfer [5,6], as also utilized by Hollmuller and Lachal [7] for the same purposes.

The suitability of one-dimensional treatment (along a radial or vertical coordinate) has been proposed and validated in a multitude of numerical and experimental studies, among which one can mention the work carried out by Tzaferis et al. [8], on eight prediction models for the performance of heat exchangers, as well as the work of Benkert et al. [9], De Paepe and Janssens [10] and Stevens [11,12]. The agreement between the model proposed here and the above publications is very satisfactory; the same is also the case as regards certain theoretical and experimental results in the literature obtained by Wagner et al. [13], Sthål [14] and Sharan and Jadhav [15].

The proposed simplified model is easier to apply (also in a spreadsheet) than other complex, albeit accurate, numerical models such as, in addition to the ones already cited [7,9,13,14], the following: Puri [16], Mihalakakou et al. [17], Jacovides et al. [18], Millette and Galanis [19], Gauthier et al. [20], Pfafferott [21]. Moreover, the analytical solution of the process of thermal exchange in EAHEs, limited up to now by a quasi-steady one-dimensional analysis which neglects the possibility of condensation in the EAHEs, for example the recent work of Shukla et al. [22], has been considerably improved. However, the very recent work of Costa [23] pointed out that the steady-state physical model is useful for a thermodynamic analysis of the overall system.

For the assessment of the performance of the EAHEs, Krarti and Kreider [24] considered that the problem is a transient one, since heat removed from air within the tube heats the surroundings earth and reduces the cooling effect relative to the steady state calculation which assumes a constant earth temperature. Their proposed analytical model assumes, after a few days of operation, the EAHEs system reaches a steady, periodic state. Also Hollmuller [25] considers a periodic input for the air in the buried pipe, yielding a physical interpretation of the amplitude-dampening and the phase-shifting of the periodic input signal. In this latter work a detailed analytical solution regarding soil perturbation around the buried pipe is proposed, but without perturbation of the free upper surface.

Indeed, the aim of the present work is to propose a transient one-dimensional analytical solution which takes into account, with some appropriate simplifications, not only the possibility of condensation in the buried pipes, but also the influence of the thermal surface wave from above the ground and the fluctuation in temperature of the air passing through the said buried pipes.

2. Theoretical basis of the model

The present model is based on the solution of the heat equation, generally referred to as the "diffusion equation", applied to ground that is considered as homogeneous and isotropic with respect to the propagation of heat.

The mathematical model that describes the physical system considered is linear and, thus, allows the application of the principle of the superposition of effects.

The solution of the heat equation, a differential equation with partial derivatives, is carried out in the domain of Laplace, through which it is generally possible to solve complex problems which it would be otherwise impossible to solve, apart from by numerical methods.

The problem in question is without internal generation of heat, hence Fourier's equation is considered:

$$\nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

It can be written in synthetic form as follows:

$$L(T) = 0 \tag{2}$$

with

$$L = \nabla^2 - \frac{1}{\alpha}\frac{\partial}{\partial t} \tag{3}$$

Operator L, that depends on the type of equation considered or rather the type of physical problem examined, satisfies the property of linearity. The superposition of causes and effects can, consequently, be applied to the linearity of operator L.

The problem in question is well-posed and, therefore, the solution depends continuously on the data, with an appropriate topology.

3. Hypothesis of the model

The proposed model is presented here pointing out its assumptions and simplifications.

The basic hypothesis of the model is a one-dimensional heat transfer, considering rectangular coordinates in plane geometry supposed with several horizontal buried pipes close to each other, so that lateral heat diffusion around the pipes is negligible in comparison to plane diffusion upwards/downwards. A similar configuration was analyzed by Wagner et al. [13] from a numerical and experimental point of view, showing a temperature field in a cross-sectional plan (x–y axis in Fig. 1). As in the major part of radial models in literature [8–10,22,24], one supposes a uniform temperature along the perimeter of the pipe surface, equal to the wall temperature calculated at $x = b$ (a good approximation as long as the pipe diameter and the thickness of the soil disturbed by the pipe are small compared to the depth of installation b of the buried pipe). As a further approximation it is possible to perform the specific analysis of this paper also for a single buried pipe, supposing, near the pipe, an axially-symmetric diffusion of heat superposed to one-dimensional heat transfer from the upper free surface. In fact, as specified in Section 2, the linearity of the system allows us to apply the principle of superposition of effects. Therefore, Fourier's equation is solved by imposing different boundary conditions (Fig. 1); by superposing these solutions in a correct way, one can determine the temperature in relation to the surface of earth to air heat exchange, in other words the wall temperature of the buried pipe, T_w. Each equation is solved by considering a transient one-dimensional propagation of heat waves. The

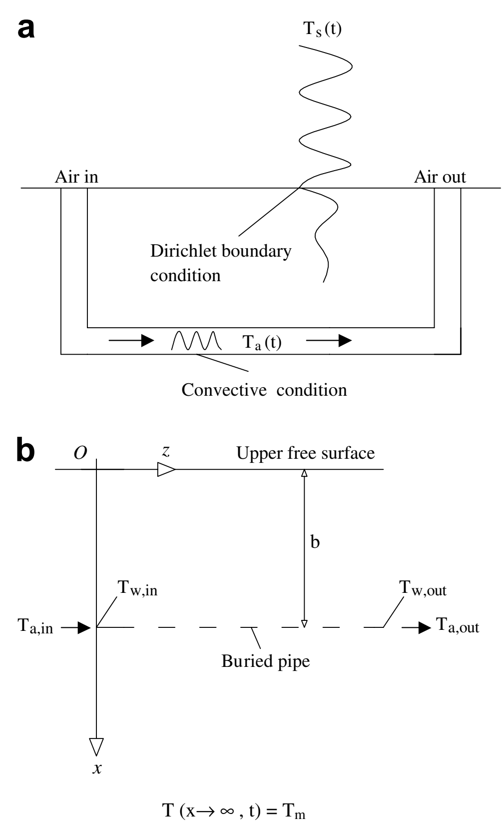

Fig. 1. Boundary conditions for the real system (a) and for the mathematical model (b).

mass transfer to take into account the moisture content in the soil as in Refs. [16,17] has not been considered; one considers uniform thermo-physical properties for the soil (α_G, k_G).

For determining the underground temperature with depth, the heat conduction is assumed to follow a quasi-steady-state behavior and the periodic solution of the Fourier's equation was used for taking into account the perturbation owing to the yearly and daily fluctuations of temperature at the free upper surface. Analogous approaches have been used in other publications as in Refs. [9,24].

These solutions were used as initial conditions for the transient solution of the flat-plate or semi-infinite body to take into account the heating in the proximity of the horizontal buried pipe in the upwards and downwards diffusion of heat. Using Green's functions, the exact solutions are yielded with the appropriate initial and boundary conditions in the operating conditions of the system (a constant mass flow rate and convective heat transfer coefficient are assumed). A different simplified solution considers a uniform initial temperature in the proximity of the buried pipe, as in some mentioned works [24], and the semi-infinite body transient solution (with convective conditions) is combined with the thermal profile in the ground applying the principle of superposition of effects.

In the considered solutions the thickness of the buried pipe and the influence of the vertical pipes at the inlet and outlet are neglected.

This one-dimensional treatment is used for determining the wall temperatures at the inlet and outlet (fixed T_{out}) of the buried pipe. Therefore, quadratic profiles are assumed for the temperatures along the z-axis and for the humidity ratio of the air layer at the buried pipe wall. These profiles are close to the experimental data and correspond to the first terms of a power series expansion, analogously to recent works in literature. With these assumptions it is possible to calculate the heat transfer rate from air to soil and to verify possible condensation. Fixing a first value of the humidity ratio at the outlet of the buried pipe W_{out} and considering the enthalpy balance on the entire buried pipe one can obtain its length L. The applicability of the convective mass transfer mechanism for treating the possible phase change for water vapor in the humid air is supposed. A saturated air layer is assumed near the pipe wall [7] and a convective mass transfer coefficient [5] is defined for obtaining the mass flow rate of condensate at the pipe surface. A differential equation is obtained whose solution furnishes the humidity ratio at the outlet of the buried pipe. An iterative method is necessary for refining the output variables W_{out} and L.

4. Effect of surface heat flux: semi-infinite body with the Dirichlet cosinusoidal condition

If a cosinusoidal temperature profile is set-up at the surface in order to obtain the profile of ground temperature, it is necessary to solve Eq. (1) with the boundary conditions:

$$\begin{aligned} &T(0,t) = T_{\text{s},m} + \Delta T_{\text{s}} \cos\left[\omega\left(t - t_{\text{s}}^*\right)\right] \\ &T(\infty,t) = T_{\text{s},m} \end{aligned} \tag{4}$$

and the solution in the time domain is [12,26]:

$$T(x,t) = T_{\text{s},m} + \Delta T_{\text{s}} e^{-\sqrt{\frac{\omega}{2\alpha}}x} \cos\left[\omega(t - t_{\text{s}}^*) - \sqrt{\frac{\omega}{2\alpha}}x\right] \tag{5}$$

This solution is used for taking into account both the yearly and daily fluctuations in the following Section 6.

5. Effect of daily fluctuation in temperature of external air passing through the buried pipe

5.1. Semi-infinite body with convective condition and constant fluid temperature $\overline{T}_{\text{a}}$

In this case Eq. (1) is solved through a convective boundary condition and constant fluid temperature $\overline{T}_{\text{a}}$. The boundary conditions are

$$\begin{aligned} &T(x,0) = T_0 \\ &T(\infty,t) = T_0 \\ &\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0} = \frac{h_{\text{c}}}{k}[T(0,t) - \overline{T}_{\text{a}}] \end{aligned} \tag{6}$$

By imposing these conditions, the obtained solution in the time domain is [26]

$$\begin{aligned} T(x,t) - T_0 = (\overline{T}_{\text{a}} - T_0)\Bigg[&\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - e^{\left(\frac{h_{\text{c}}x}{k} + \frac{h_{\text{c}}^2\alpha t}{k^2}\right)} \\ &\cdot \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h_{\text{c}}\sqrt{\alpha t}}{k}\right)\Bigg] \end{aligned} \tag{7}$$

where by definition

$$\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) = 1 - \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) \tag{8}$$

This solution is used in the following Section 6.2 for determining the influence of the buried pipe in the ground, taking care to translate the vertical coordinate in $|x - b|$, considering the installation depth of the buried pipe.

5.2. Semi-infinite body with convective condition and cosinusoidal fluid temperature

This equation has a boundary condition of a convective type similar to that considered in Section 5.1. This time, however, the temperature of the fluid is not constant, but rather cosinusoidal:

$$\begin{aligned} &T(\infty,t) : \text{limited} \\ &-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h_{\text{c}}[T_{\text{a}}(t) - T(0,t)] \end{aligned} \tag{9}$$

where

$$T_{\text{a}}(t) = T_{\text{a,d}} + \Delta T_{\text{a}} \cos\left[2\pi\left(\frac{t - t_{\text{a}}^*}{t_0}\right)\right] \tag{10}$$

The solution in the time domain is [12,26]

$$T(x,t) = T_{\mathrm{a,d}} + \frac{\Delta T_{\mathrm{a}} \mathrm{e}^{-x\sqrt{\left(\frac{\omega_{\mathrm{d}}}{2\alpha}\right)}}}{\sqrt{1+2\beta+2\beta^2}} \cos\left[\omega_{\mathrm{d}}(t-t_{\mathrm{a}}^*) - x\sqrt{\frac{\omega_{\mathrm{d}}}{2\alpha}} - \varepsilon\right] \tag{11}$$

with

$$\omega_{\mathrm{d}} = \frac{2\pi}{t_0}, \quad \beta = \frac{k}{h_{\mathrm{c}}}\left(\frac{\pi}{\alpha t_0}\right)^{\frac{1}{2}}, \quad \varepsilon = tg^{-1}\left(\frac{\beta}{\beta+1}\right) \tag{12}$$

This solution is considered in the following Section 6.2 to take into account the daily fluctuations of the air temperature passing through the buried pipe in the continuous regime of operation.

5.3. Solution of semi-infinite body with convective conditions and initial temperature $f(x)$

The solution of the Fourier equation with initial condition $T_{\mathrm{G}}(x,0)=f(x)$ and convective boundary conditions with a fluid temperature varying (in time) with law $\phi(t)$, is obtained through the use of Green's functions (u) to give the following expression [2,26]:

$$\begin{aligned} T(x,t) &= \int f(x')u|_{t'=0}dx' + \alpha\int \phi(t')\frac{\partial u}{\partial x'}\bigg|_{x'=0} \mathrm{d}t' \\ u(x,t|x',t') &= \frac{1}{2\sqrt{\pi\alpha t}}\left[\mathrm{e}^{-(x-x')^2}/4\alpha(t-t') + \mathrm{e}^{-(x+x')^2/4\alpha(t-t')}\right. \\ &\quad - \frac{h_{\mathrm{c}}}{k}\mathrm{e}^{\alpha\frac{h_{\mathrm{c}}^2}{k^2}(t-t')+\frac{h_{\mathrm{c}}}{k}(x+x')}\mathrm{erfc}\left[\frac{x+x'}{2\sqrt{\alpha(t-t')}}\right. \\ &\quad \left.\left. + \frac{h_{\mathrm{c}}}{k}\sqrt{\alpha(t-t')}\right]\right] \end{aligned} \tag{13}$$

with

$$\frac{\partial u}{\partial x}\bigg|_{x=0} = \frac{h_{\mathrm{c}}}{k}u\bigg|_{x=0}$$

This solution is considered in Section 6.1 to evaluate the heat transfer in the ground under the buried pipe.

5.4. Flat plate, transient 1D: Dirichlet boundary conditions with law $\phi_1(t)$ at $x = 0$, convection with law $\phi_2(t)$ at $x = b$ and initial temperature $f(x)$ [2,26]

In this case

$$\begin{aligned} T(x,t) &= \int f(x')u|_{t'=0}\mathrm{d}x' + \alpha\int \phi_1(t')\frac{\partial u}{\partial x'}\bigg|_{x'=0} \mathrm{d}t' \\ &\quad + \alpha\int \phi_2(t')\frac{h_{\mathrm{c}}}{k}u\bigg|_{x'=b} \mathrm{d}t' \\ u(x,t|x',t') &= \frac{2}{b}\sum_{m=1}^{\infty} \exp\left[-\frac{\beta_m^2\alpha(t-t')}{b^2}\right]\left(\frac{\beta_m^2+Bi}{\beta_m^2+Bi^2+Bi}\right) \\ &\quad \times \sin\left(\beta_m\frac{x}{b}\right)\sin\left(\beta_m\frac{x'}{b}\right) \end{aligned} \tag{14}$$

with

$$\beta_m \cot\beta_m = -Bi = -h_{\mathrm{c}}b/k \quad \text{and} \quad \frac{\partial u}{\partial x}\bigg|_{x=b} = -\frac{h_{\mathrm{c}}}{k}u\bigg|_{x=b}$$

This solution is considered in Section 6.1 to evaluate the heat transfer in the ground between the upper free surface and the buried pipe.

5.5. Flat plate, transient 1D: convection with law $\phi_1(t)$ at $x = 0$, Neumann ($\partial u/\partial x = 0$) at $x = B$ and initial temperature $f(x)$ [2,26]

In this conditions:

$$\begin{aligned} T(x,t) &= \int f(x')u|_{t'=0}\mathrm{d}x' + \alpha\int \phi_1(t')\frac{h_{\mathrm{c}}}{k}u\bigg|_{x'=0} \mathrm{d}t' \\ u(x,t|x',t') &= \frac{2}{B}\sum_{m=1}^{\infty} \exp\left[-\frac{\beta_m^2\alpha(t-t')}{B^2}\right]\left(\frac{\beta_m^2+Bi}{\beta_m^2+Bi^2+Bi}\right) \\ &\quad \times \cos\left[\beta_m\left(1-\frac{x}{B}\right)\right]\cos\left[\beta_m\left(1-\frac{x'}{B}\right)\right] \end{aligned} \tag{15}$$

with

$$\beta_m \tan\beta_m = Bi = h_{\mathrm{c}}B/k \quad \text{and} \quad \frac{\partial u}{\partial x}\bigg|_{x=0} = \frac{h_{\mathrm{c}}}{k}u\bigg|_{x=0}$$

This solution is considered in Section 6.1 to evaluate the heat transfer between the depth of installation of the buried pipe and a wide depth in the ground at constant temperature.

6. Temperature at the surface of the buried pipe

6.1. Analytical solution

Through the principle of the superposition of causes and effects it is possible to arrive at the law of temperature variation of the ground on a given day by adding the solutions of the Fourier equation to Dirichlet boundary condition at the surface with cosinusoidal law, calculated on a yearly and a daily basis, respectively; this gives

$$T_{\mathrm{G}}(x,t) = T_{\mathrm{G,I}}(x,t) + T_{\mathrm{G,II}}(x,t) \tag{16}$$

By utilizing Eq. (4) with a value of $T_{\mathrm{s},m}$ equal to the average annual temperature in relation to the ground surface, constant in the interval considered (yearly), from the application of Eq. (5) on the yearly basis the term $T_{\mathrm{G,I}}(x,t)$, which appears in Eq. (16), is determined; this expresses the influence of annual temperature fluctuations at different depths, thus

$$T_{\mathrm{G,I}}(x,t) = T_{\mathrm{s,y}} + \Delta T_{\mathrm{s,y}}\mathrm{e}^{-\sqrt{\frac{\omega_{\mathrm{y}}}{2\alpha}}x}\cos\left[\omega_{\mathrm{y}}(t+\tau-\tau_0) - \sqrt{\frac{\omega_{\mathrm{y}}}{2\alpha}}x\right] \tag{17}$$

It should be noted that the term $T_{s,y}$ is assumed to be constant; it represents the temperature of the ground at greater depths (over 5–6 m) and remains unchanged throughout the year.

For the calculation of daily fluctuations at the surface Eq. (5) can be applied on a daily basis to give

$$T_{G,II}(x,t) = \Delta T_{s,d} e^{-\sqrt{\frac{\omega_d}{2\alpha}}x} \cos\left[\omega_d(t - t_s^*) - \sqrt{\frac{\omega_d}{2\alpha}}x\right] \tag{18}$$

Eq. (18) is obtained by imposing $T_{s,m} = 0$ in Eq. (5), as the constant value of temperature applied on the surface ($T_{s,m}$) has already been considered in Eq. (17). The correct application of the principle of the superposition of effects, in fact, requires that every effect is superposed once only.

For evaluating the influence of annual and daily fluctuations in surface temperature on ground temperature at different depths, therefore, it is possible to add up the effects, utilizing the principle of the superposition of causes and effects:

$$\begin{aligned} T_G(x,t) &= T_{G,I}(x,t) + T_{G,II}(x,t) \\ &= T_{s,y} + \Delta T_{s,y} e^{-\sqrt{\frac{\omega_y}{2\alpha}}x} \cos\left[\omega_y(t+\tau-\tau_0) - \sqrt{\frac{\omega_y}{2\alpha}}x\right] \\ &\quad + \Delta T_{s,d} e^{-\sqrt{\frac{\omega_d}{2\alpha}}x} \cos\left[\omega_d\left(t - t_s^*\right) - \sqrt{\frac{\omega_d}{2\alpha}}x\right] \end{aligned} \tag{19}$$

Through the application of Eq. (18), on a daily basis, it is possible to ascertain that the influence of daily fluctuations even at a depth of 0.5–1 m is negligible, even if it is still considered a good idea to add up its contribution in Eq. (16) in order to have as full and comprehensive a description as possible.

If b represents the depth of the installation of the buried pipe and x is into the interval $[0, b]$, one can suppose the boundary conditions of Section 5.4, by utilizing the Eq. (14) with $f(x) = T_G(x,t_1)$ and $\phi_1(t) = T_s(t) = T_G(0,t)$ from Eq. (19), $\phi_2(t) = T_a(t)$ from Eq. (10), to obtain the following:

$$\begin{aligned} T(x,t) &= \int_0^b T_G(x',t_1) \cdot u(x, t-t_1|x',0)\mathrm{d}x' \\ &\quad + \alpha \int_0^{t-t_1} T_s(t'+t_1) \frac{\partial u}{\partial x'}\bigg|_{x'=0} \mathrm{d}t' \\ &\quad + \alpha \int_0^{t-t_1} T_a(t'+t_1) \frac{h_c}{k} u\bigg|_{x'=b} \mathrm{d}t' \end{aligned} \tag{20}$$

Indeed, air subject to variable temperature in time, according to the cosinusoidal law, passes through the buried pipe at a depth of $x = b$, from instant t_1. In interval $(b, +\infty)$, on the basis of the considerations made in the previous section, by utilizing Eq. (13), by substituting $f(x) = T_G(x,t_1)$ from Eq. (19) and $\phi(t) = T_a(t)$ from Eq. (10), one arrives at the following solution:

$$\begin{aligned} T(x,t) &= \int_b^\infty T_G(x',t_1) \cdot u(x-b, t-t_1|x'-b, 0)\mathrm{d}x' \\ &\quad + \alpha \int_0^{t-t_1} T_a(t'+t_1) \frac{h_c}{k} u(x-b, t-t_1|0,t')\mathrm{d}t' \end{aligned} \tag{21}$$

It is not necessary to evaluate the integral of Eq. (21) in the range $(b, +\infty)$, as in Ref. [26], but up to a depth at constant temperature (about 20 m), utilizing the solution of Section 5.5. Eqs. (20) and (21) can be solved by numerical integration methods, which have the obvious advantage of providing a transient solution for the ground temperature profile without the use of simplified hypothesis.

On the basis of this solution, one can obtain values for the wall temperature of the buried pipe T_w when the system is operating.

6.2. Simplified solution

The analytical solution, however, requires a high computational workload, so a more immediate, simplified method, which provides very similar results to the analytical solution, is proposed.

The wall temperature of the buried pipe, assumed equal to that of the ground at the same level (hypothesis of the negligible resistance of the buried pipe) can be obtained by the superposing of the different solutions already obtained (see Sections 4, 5.1 and 5.2). It is necessary, therefore, to consider both the geometry of the physical system analyzed and the boundary conditions imposed for the solution of each equation.

On the basis of the correct hypothesis, reported further on, temperature $T(x,t)$ is given by the combination of four contributions, two of which are linked to the fluctuation in surface temperature and two linked to earth to air heat exchange:

$$\begin{aligned} T(x,t) &= T_G(x,t) + T_{HE}(x,t) \\ &= T_{G,I}(x,t) + T_{G,II}(x,t) + T_{HE,I}(x,t) + T_{HE,II}(x,t) \end{aligned} \tag{22}$$

It is preferable to pursue an approximate approach for the determination of the influence of earth to air heat exchange; the term $T_{HE}(x,t)$ can be added to the term $T_G(x,t)$, calculated through Eq. (19), in order to assess the effective temperature of the ground.

The term $T_{HE}(x,t)$ depends on the earth to air heat exchange on the surface of the buried pipe, with air temperature $\overline{T}_a$ constant in time interval $[t_1, t]$ and Eq. (7) can be used for this purpose. An initial constant condition is assumed $T_0 = T_G(b,t_1)$, to ensure that the effect owing to the air temperature $\overline{T}_a$ takes into account the real initial conditions for $t > t_1$. In this way, one can obtain the solution for the effective earth to air heat exchange, making sure to cancel the first term T_0, linked exclusively to the initial condition, from Eq. (7). This step is necessary in order to avoid the superposition of this value for a second time, thereby breaking the principle of the superposition of effects. Moreover, before Eq. (7) can be inserted in Eq. (22) to calculate term $T_{\mathrm{HE,I}}(x,t)$, in order to use the same independent variable x for all its terms, it is necessary to translate the coordinates of Eq. (7) in the following way:

$$T_{\mathrm{HE,I}}(x,t)=\left[\overline{T}_{\mathrm{a}}-T_{\mathrm{G}}(b,t_1)\right]\left\{\operatorname{erfc}\left(\frac{|b-x|}{2\sqrt{\alpha(t-t_1)}}\right)-\mathrm{e}^{\left[\frac{h_{\mathrm{c}}|b-x|}{k}+\frac{h_{\mathrm{c}}^2\alpha(t-t_1)}{k^2}\right]}\cdot\operatorname{erfc}\left(\frac{|b-x|}{2\sqrt{\alpha(t-t_1)}}+\frac{h_{\mathrm{c}}\sqrt{\alpha(t-t_1)}}{k}\right)\right\} \tag{23}$$

where the average (constant) value of the temperature of the air through the buried pipe in time interval $[t_1,t]$, calculated by taking into account the boundary conditions in Eq. (10), is

$$\overline{T}_{\mathrm{a}}=\frac{1}{t-t_1}\int_{t_1}^{t}T_{\mathrm{a}}(t')\mathrm{d}t'=T_{\mathrm{a,d}}+\frac{\Delta T_{\mathrm{a}}}{\omega_{\mathrm{d}}(t-t_1)}\left\{sen\left[\omega_{\mathrm{d}}(t-t_{\mathrm{a}}^*)\right]-sen\left[\omega_{\mathrm{d}}(t_1-t_{\mathrm{a}}^*)\right]\right\} \tag{24}$$

The term $T_{\mathrm{HE,II}}(x,t)$ in Eq. (22) is linked to the daily fluctuation of air through the buried pipe and is considered only in the case of the continuous working of the system (for a wide multiple of the entire period $2\pi/\omega_{\mathrm{d}}$); this term derives from Eq. (11), which, for the same reasons outlined above, is thus reformulated:

$$T_{\mathrm{HE,II}}(x,t)=\frac{\Delta T_{\mathrm{a}}\mathrm{e}^{-|b-x|\sqrt{\left(\frac{\omega_{\mathrm{d}}}{2\alpha}\right)}}}{\sqrt{1+2\beta+2\beta^2}}\cos\left[\omega_{\mathrm{d}}(t-t_{\mathrm{a}}^*)-|b-x|\sqrt{\left(\frac{\omega_{\mathrm{d}}}{2\alpha}\right)}-\varepsilon\right] \tag{25}$$

By replacing Eqs. (19), (23) and (25) in Eq. (22), the following result is achieved:

$$T(x,t)=T_{\mathrm{G}}(x,t)+\left[\overline{T}_{\mathrm{a}}-T_{\mathrm{G}}(b,t_1)\right]\cdot\left\{\operatorname{erfc}\left[\frac{|b-x|}{2\sqrt{\alpha(t-t_1)}}\right]-\mathrm{e}^{\left[\frac{h_{\mathrm{c}}|b-x|}{k}+\frac{h_{\mathrm{c}}^2\alpha(t-t_1)}{k^2}\right]}\cdot\operatorname{erfc}\left[\frac{|b-x|}{2\sqrt{\alpha(t-t_1)}}+\frac{h_{\mathrm{c}}\sqrt{\alpha(t-t_1)}}{k}\right]\right\}+\frac{\Delta T_{\mathrm{a,d}}\mathrm{e}^{-|b-x|\sqrt{\frac{\omega_{\mathrm{d}}}{2\alpha}}}}{\sqrt{1+2\beta+2\beta^2}}\cos\left[\omega_{\mathrm{d}}(t-t_{\mathrm{a}}^*)-|b-x|\sqrt{\frac{\omega_{\mathrm{d}}}{2\alpha}}-\varepsilon\right] \tag{26}$$

The profile of $T(x,t)$ derived from Eq. (26) is illustrated for a particular day of the year in Figs. 2 and 3, for two different depths of the buried pipe. It should be noted that, at a depth of more than 1 m, the influence of the daily thermal wave is negligible; the influence of the surface thermal wave, on the other hand, evaluated on a yearly basis becomes negligible from a depth of 5 to 6 m.

The comparison between the analytical solution obtained with Green's functions and the simplified method is illustrated in Fig. 4. Over around 200 simulations, 24 of which are reported in Table 1, the average percent error

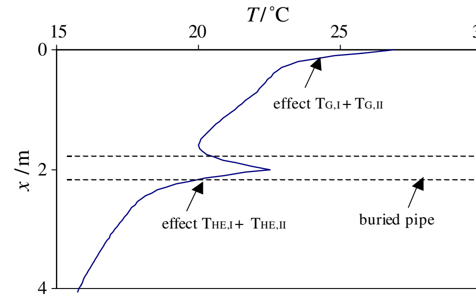

Fig. 2. Ground temperature profile obtained by the simplified model ($b=2$ m, $d_{\mathrm{h}}=0.15$ m, $\dot{m}=0.067$ kg/s, $\alpha_{\mathrm{G}}=1.0\times10^{-6}$ m^2/s, $k_{\mathrm{G}}=2$ W/m K, $T_{\mathrm{s,y}}=15$ °C, $\Delta T_{\mathrm{s,y}}=9$ °C, $\Delta T_{\mathrm{s,d}}=3$ °C, $T_{\mathrm{a,d}}=21$ °C, $\Delta T_{\mathrm{a,d}}=9$ °C, $\tau=\tau_0$, $t_1=7$ h, $t=t_{\mathrm{a}}^*=15$ h, $t_{\mathrm{s}}^*=16$ h).

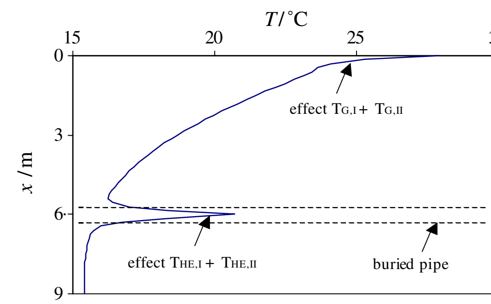

Fig. 3. Ground temperature profile obtained by the simplified model ($b=6$ m, $d_{\mathrm{h}}=0.53$ m, $\dot{m}=1$ kg/s, $\alpha_{\mathrm{G}}=1.3\times10^{-6}$ m^2/s, $k_{\mathrm{G}}=2$ W/m K, $T_{\mathrm{s,y}}=16$ °C, $\Delta T_{\mathrm{s,y}}=9$ °C, $\Delta T_{\mathrm{s,d}}=3$ °C, $T_{\mathrm{a,d}}=21$ °C, $\Delta T_{\mathrm{a,d}}=9$ °C, $\tau=\tau_0$, $t_1=7$ h, $t=t_{\mathrm{a}}^*=15$ h, $t_{\mathrm{s}}^*=16$ h).

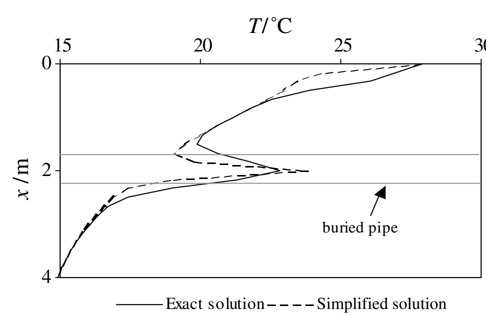

Fig. 4. Exact and simplified ground temperature profiles ($b=2$ m, $d_{\mathrm{h}}=0.53$ m, $\dot{m}=1.67$ kg/s, $\alpha_{\mathrm{G}}=0.6\times10^{-6}$ m^2/s, $k_{\mathrm{G}}=2$ W/m K, $T_{\mathrm{s,y}}=15$ °C, $\Delta T_{\mathrm{s,y}}=10$ °C, $\Delta T_{\mathrm{s,d}}=3$ °C, $t_{\mathrm{s}}^*=15$ h, $T_{\mathrm{a,d}}=23$ °C, $\Delta T_{\mathrm{a,d}}=10$ °C, $\tau-\tau_0=0$, $t=14$ h, $t_1=8$ h, $t_{\mathrm{a}}^*=14$ h).

between the temperatures obtained by the two methods, applied in relation to the temperature at the installation depth of the buried pipe is below around 5% and in the proximity of the installation is always below 12%.

Finally, we come to the determination of the temperature of the buried pipe, an indispensable factor in calculating the quantity of heat exchanged by the air passing through the buried pipe and the ground. The wall temperature of the buried pipe can be obtained assuming $x=b$ in Eq. (26); we thus have

Table 1
Comparison between the exact and simplified profile of temperature in the ground (other parameters: $d_h = 0.53$ m, $\dot{m} = 1.67$ kg/s, $\alpha_G = 0.6 \times 10^{-6}$ m^2/s, $k_G = 2$ W/m K, $T_{s,y} = 15$ °C, $\Delta T_{s,y} = 10$ °C, $\Delta T_{s,d} = 3$ °C, $t_s^* = 15$ h, $\Delta T_{a,d} = 10$ °C, $\tau - \tau_0 = 0$, $t = 14$ h, $t_1 = 8$ h, $t_a^* = 14$ h)

Simulation data		Temperature by simplified solution (°C)	Temperature by exact solution (°C)	Percent error[a] (%)
$T_{a,d} = 18$ °C	$x = 1.83$ m	19.234	21.213	9.33
	$x = b = 2$ m	21.283	22.076	3.59
	$x = 2.17$ m	18.384	20.749	11.40
$T_{a,d} = 23$ °C	$x = 0.92$ m	23.630	24.880	5.02
	$x = b = 1$ m	25.336	26.070	2.82
	$x = 1.08$ m	23.037	25.374	9.21
$T_{a,d} = 23$ °C	$x = 1.83$ m	19.811	21.739	8.87
	$x = b = 2$ m	23.835	22.911	4.03
	$x = 2.17$ m	18.960	21.371	11.28
$T_{a,d} = 23$ °C	$x = 3.67$ m	15.303	16.524	7.39
	$x = b = 4$ m	19.734	18.933	4.23
	$x = 4.33$ m	14.794	16.066	7.92
$\dot{m} = 0.5$ kg/s	$x = 1.83$ m	19.142	20.498	6.62
	$x = b = 2$ m	21.104	20.419	3.35
	$x = 2.17$ m	18.292	19.601	6.68
$\dot{m} = 1$ kg/s	$x = 1.83$ m	19.490	21.232	8.20
	$x = b = 2$ m	22.584	21.508	5.00
	$x = 2.17$ m	18.640	20.452	8.86
$\alpha = 1 \times 10^{-6}$ m^2/s	$x = 1.83$ m	21.738	23.358	6.94
	$x = b = 2$ m	25.157	24.816	1.37
	$x = 2.17$ m	20.958	23.484	10.76
$k_G = 1.5$ W/m K	$x = 1.83$ m	20.060	22.036	8.97
	$x = b = 2$ m	24.729	23.491	5.27
	$x = 2.17$ m	19.210	21.785	11.82

[a] $100 \cdot |\text{Simplified} - \text{Exact}|/\text{Exact}$.

$$\begin{aligned} T_w &= T(b,t) \\ &= T_G(b,t) + [\overline{T}_a - T_G(b,t_1)] \\ &\quad \cdot \left\{1 - e^{\left[\frac{h_c^2 \alpha (t-t_1)}{k^2}\right]} \cdot \operatorname{erfc}\left[\frac{h_c\sqrt{\alpha(t-t_1)}}{k}\right]\right\} \\ &\quad + \frac{\Delta T_{a,d}}{\sqrt{1+2\beta+2\beta^2}} \cos[\omega_d(t-t_a^*) - \varepsilon] \end{aligned} \tag{27}$$

The above relation takes into account both the effect due to the wave of heat from the surface of the ground and also the daily fluctuation of temperature in the external air that passes through the buried pipe. As the solution used is a transient one-dimensional one, Eq. (27) does not supply the evolution of the temperature of the side in direction z (direction of the flow of air). In order to calculate the degree of heat exchange between the air and the buried pipe, one can hypothesize that the wall temperature of the buried pipe remains constant along z or varies following a pre-established law.

7. Performance of earth-to-air heat exchangers

In order to determine the performances of EAHEs it is necessary to establish the geometry of the buried pipe, the thermo-physical (constant) properties of the ground and air and the thermo-hygrometric conditions of the air at the inlet of the buried pipe and the air mass flow rate. Generally turbulent conditions occur and therefore, the heat transfer coefficient h_c could be calculated by the Dittus–Boelter correlation [27,28]. If, moreover, fixed a value of the average temperature at the outlet T_{out}, obviously compatible with the thermo-hygrometric conditions of the ambient air to be cooled, the unknown factors of the problem remain the length of the buried pipe L and the specific humidity of the air at the outlet. Quadratic laws can be assumed for the profiles of the temperature along coordinate z. This hypothesis allows us to evaluate more precisely the performance of the buried pipes, but it implies a greater complexity in the obtained correlations.

7.1. Case of heat transfer without mass transfer

Through Eq. (27) it is possible to calculate the wall temperature of the buried pipe in the inlet section $T_{w,in}$ and the outlet section $T_{w,out}$; in the latter case, considering that T_{out} can be assumed to be constant in the time interval considered $[t_1, t]$, one can write

$$T_{w,out} = T_G(b,t) + [T_{out} - T_G(b,t_1)]\left\{1 - e^{\left[\frac{h_c^2 \alpha (t-t_1)}{k^2}\right]} \cdot \operatorname{erfc}\left[\frac{h_c\sqrt{\alpha(t-t_1)}}{k}\right]\right\} \tag{28}$$

The presence of condensation in the buried pipe must be verified, calculating the humidity ratio of air at the wall temperature in saturated conditions at the inlet and outlet of the pipe ($W_{R,in}$ and $W_{R,out}$) and the humidity ratio corresponding to the inlet of the air (W_{in}). If $W_{in} < W_{R,in}$ it is possible to neglect the condensation (also the evaporation is neglected for the reasons treated in the following subsection). In this case the use of the mass transfer mechanism it is not necessary for analyzing the phase change for water vapor in the humid air. An exponential longitudinal profile is obtained for the air temperature as a solution to the balance of sensible energy in the buried pipe with isothermal conditions at its surface as in Refs. [10,21,22,24,29] or at an appropriate radial distance from the pipe as in Ref. [25].

As examined in this paper, the wall temperature is not constant along the z-axis and its initial conditions are influenced by the perturbation from the upper free surface. Moreover, in the case of phase change along the length of the buried pipe the latent heat exchange must be considered. The real profile is therefore more complex than the profile used in the references cited above.

7.2. Case of heat transfer with condensation

If $W_{in} > W_{R,in}$ the condensations are not negligible. In this case, the approach of Boulama et al. [30] is preferred considering the first terms of a power series expansion for the temperatures and for the humidity ratio of the air layer near the wall of the buried pipe. An isothermal boundary condition is not assumed for the wall temperature and therefore, an exponential profile is not preferred, but quadratic profiles are used. These profiles are close to the experimental data and correspond to all the boundary conditions for the inlet and outlet of the air.

Assuming

$$T^*(z) = T_a(z) - T_w(z) \tag{29}$$

by considering a parabolic law for T_a and for T_w along coordinate z (the law that best interpolates the experimental data), one can assume a quadratic law also for $T^*(z)$, as follows:

$$T^*(z) = T^*_{in} - 2\frac{T^*_{in} - T^*_{out}}{L}z + \frac{T^*_{in} - T^*_{out}}{L^2}z^2 \tag{30}$$

From this hypothesis it is possible to determine the total instantaneous rate of heat assumed exchanged only by convection between the air and the buried pipe, obtaining in consequence:

$$\dot{Q} = \int_0^L h_c P[T_a(z) - T_w(z)]dz = \int_0^L h_c P T^*(z)dz$$
$$= h_c P\left(\frac{2}{3}T^*_{out} + \frac{T^*_{in}}{3}\right)L \tag{31}$$

In fact, the value of $\dot{Q}$ corresponds to the enthalpy variation of humid air, as regards enthalpy between the inlet and outlet sections of the buried pipe, if the enthalpy of the condensed vapor is neglected; we get by Eq. (31):

$$L = \frac{3\dot{m}_a(h_{in} - h_{out})}{h_c P(2T^*_{out} + T^*_{in})} \tag{32}$$

For calculating h_{out} it is necessary to calculate W_{out}. One can utilize a method proposed by Cucumo et al. in Ref. [4], which supplies, as obtained further down, the following relation:

$$\begin{cases} W_{out} = (W_{in} - W_{R,in})e^{-a} + (W_{R,in} - W_{R,out}) \cdot b + W_{R,in} \\ W_{in} > W_{R,in} \end{cases} \tag{33}$$

with

$$a = \frac{h_m PL}{\dot{m}_a} \cong \frac{h_c PL}{\dot{m}_a c_p} = \text{NTU} \quad \text{and}$$
$$b = \frac{2(1 - e^{-a})}{a^2} - \frac{2e^{-a}}{a} - 1 \tag{34}$$

It is evident that for $W_{in} \leqslant W_{R,in}$ it is possible to exclude any condensation that might occur and one can, therefore, assume $W_{out} = W_{in}$. One can also exclude, in the conditions under consideration, the possibility of evaporation as the buried pipes are impermeable and are supplied with a drainage system for removing liquid condensation. Eq. (33) has been obtained from the first-order differential equation with constant coefficients derived from the equality of the amount of condensation in an infinitesimal element of buried pipe dz and the same value obtained by utilizing the definition of the mass transfer coefficient h_m [5], analogous to Ref. [30]:

$$\dot{m}_a \, dW_a = h_m[W_R(z) - W_a(z)]P \, dz \tag{35}$$

The mass transfer coefficient h_m can be obtained through the heat and mass transfer analogy [27] or, for simplicity, through the Lewis relation [6,7].

For $W_R(z)$, the humidity ratio in the saturated air layer, considered at the wall temperature of the buried pipe, for the same reasons outlined above, a quadratic law of the following type was assumed:

$$W_R(z) = W_{R,in} - 2\frac{W_{R,in} - W_{R,out}}{L}z + \frac{W_{R,in} - W_{R,out}}{L^2}z^2 \tag{36}$$

Other longitudinal profiles for W_R are not considered, because, in general, one can assume that W_R depends on the temperature with a quadratic law and consequently, one prefers to use the first terms of a power series expansion with appropriate boundary conditions at the inlet and outlet of the pipe. This profile is close to the predicted values by a numerical code [3], as shown in the following Section 8.

From the iterative solutions of Eqs. (32) and (33), expressing L and W_{out}, respectively we obtain the dimensioning of the buried pipe. Eqs. (32) and (33) can be, moreover, utilized for the determination of thermo-hygrometric parameters of the air at the outlet of a heat exchanger of known length in order to estimate its performance and to carry out an assessment of the condition of the air coming out of buried pipes.

8. Validation of results

The validity of Eqs. (32) and (33) has been demonstrated through a numerical code based on the finite-difference method reported in Ref. [3], with a complete discretization for the soil and the buried pipe. We created a data-base, running the code in the most disparate input conditions, in particular ranging $\dot{m}$ from 200 m^3/h to 3200 m^3/h, the $T_{a,in}$ from 28 to 36 °C, the $T_{a,out}$ from 23 to 27 °C and the $T_{s,y}$ from 15 to 20 °C. Roughly 5000 simulations have been performed and, for buried pipes of length less than 50 m, 90% of the cases, the maximum percentage error is below 10%, as can be seen in Fig. 5, where the lengths obtained by the numerical code are reported on the abscissa axis and the lengths obtained by the analytical model on the ordinate axis, with the same input conditions for both.

Fig. 6 reports the comparison of the profiles of air temperature between inlet and outlet, obtained through the proposed model and certain experimental data obtained from Sharan and Jadhav [15]. In this figure the hypothesis of quadratic law is compared with some experimental data, but the outlet temperature which appears in this profile from Eq. (30) must be calculated by the model, with the

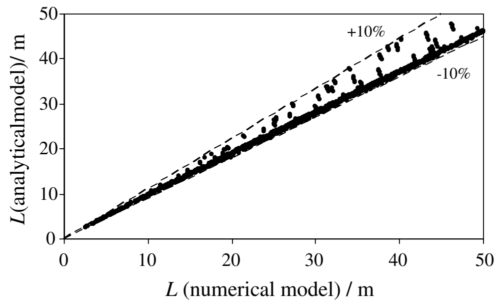

Fig. 5. Comparison between the length obtained through a numerical code [3] and the analytical model.

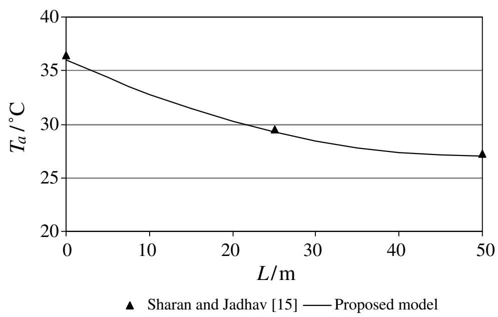

Fig. 6. Comparison between temperature profiles obtained by the proposed model and certain experimental data from [15] ($b = 3$ m, $d = 0.10$ m, $L = 50$ m, $\dot{m} = 0.1$ kg/s, $\alpha_G = 1{,}0 \times 10^{-6}$ m^2/s, $k_G = 2.7$ W/m K, $T_{s,y} = 25.7$ °C, $\Delta T_{s,y} = 5$ °C, $\Delta T_{s,d} = 3$ °C, $T_{a,d} = 27$ °C, $\Delta T_{a,d} = 13$ °C, $\tau - \tau_0 = 0$, $t - t_1 = 2$ h, $t_a^* = 14$ h).

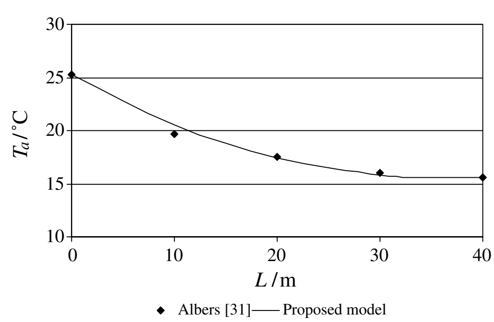

Fig. 7. Comparison between temperature profiles obtained from the proposed model and certain experimental data from [31] ($b \cong 1.25$ m, $d = 0.125$ m, $L = 40$ m, $\dot{m} = 0.077$ kg/s, $\alpha_G = 0.72 \times 10^{-6}$ m^2/s, $k_G = 1.5$ W/m K, $T_{s,y} = 12.8$ °C, $\Delta T_{s,y} = 0$ °C, $\Delta T_{s,d} = 0$ °C, $T_{a,d} = 25.3$ °C, $\Delta T_{a,d} = 0$ °C).

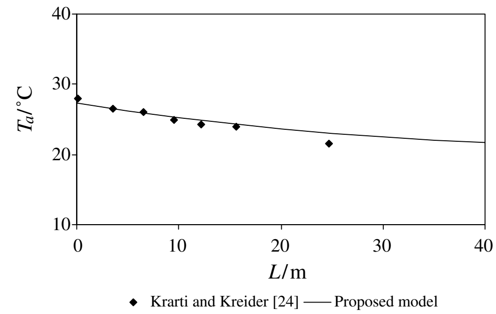

Fig. 8. Comparison between temperature profiles obtained from the proposed model and certain experimental data from [24] ($b = 2.286$ m, $d = 0.305$ m, $L = 40$ m, $\dot{m} = 0.13$ kg/s, $\alpha_G = 6.45 \times 10^{-7}$ m^2/s, $k_G = 1.16$ W/m K, $T_{s,y} = 17.4$ °C, $\Delta T_{s,y} = 10$ °C, $\Delta T_{s,d} = 3$ °C, $T_{a,d} = 25.5$ °C, $\Delta T_{a,d} = 7$ °C, $t_s^* = 15$ h, $\tau - \tau_0 = 0$, $t = 20$ h, $t_1 = 12$ h, $t_a^* = 13$ h).

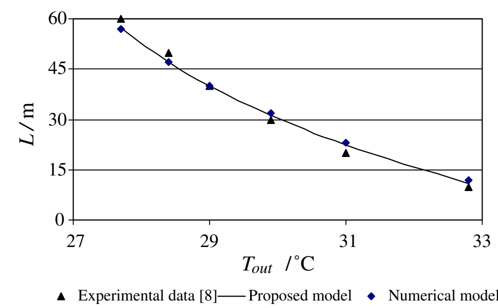

Fig. 9. Length of buried pipe in relation to the temperature at the outlet of air ($b = 1.5$ m, $d = 0.125$ m, $\dot{m} = 0.3$ kg/s, $\alpha_G = 0.6 \times 10^{-6}$ m^2/s, $k_G = 2$ W/m K, $T_{s,y} = 20$ °C, $\Delta T_{s,y} = 10$ °C, $\Delta T_{s,d} = 3$ °C, $T_{a,d} = 35$ °C, $\Delta T_{a,d} = 0$ °C).

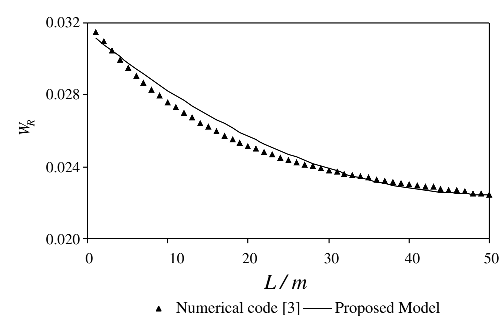

Fig. 10. Comparison between the profiles of humidity ratio obtained from the proposed model and certain numerical data from [3] at the wall temperature ($b = 3$ m, $d = 0.10$ m, $L = 50$ m, $\dot{m} = 0.1$ kg/s, $\alpha_G = 1.0 \times 10^{-6}$ m^2/s, $k_G = 2.7$ W/m K, $T_{s,y} = 25.7$ °C, $\Delta T_{s,y} = 5$ °C, $\Delta T_{s,d} = 3$ °C, $T_{a,d} = 27$ °C, $\Delta T_{a,d} = 13$ °C, $\tau - \tau_0 = 0$, $t - t_1 = 2$ h, $t_a^* = 14$ h, $W_{R,in} = 0.03$).

input conditions illustrated in the legend for the fixed length.

In analogous manner, Figs. 7 and 8 report the comparison with the experimental data from Albers [31] and Krarti and Kreider [24].

In relation to preset lengths of the buried pipes, Fig. 9 reports the air temperature at the outlet, calculated through the model and comparing it with experimental data from Tzaferis et al. [8], for a circular buried pipe, at equal conditions.

A comparison between the profile of the humidity ratio along the pipe at the wall temperature with the parameters obtained by the proposed model and the profile obtained by a numerical code [3] was reported in Fig. 10, assuming the same input conditions for both.

On the other hand, in relation to the relative humidity of the air coming out of the buried pipe, experimental measurements by Wagner et al. [13] have shown that the average relative humidity at the outlet is above 80% in only 10% of the cases considered. This result is in line with the results of the proposed model given the same input conditions.

9. Conclusion

This paper proposed a transient one-dimensional analytical model that can be utilized for the correct installation and the calculation of the performances of EAHEs. In this way it is possible to take into account:

- the contribution of heat from the ground surface;
- the overheating of the pipe wall;
- the latent heat exchanges in the buried pipe.

With appropriate hypothesis, the model allows us to calculate the length and humidity ratio at the outlet section of a buried pipe, at an arbitrary depth of installation, assigned thermo-physical properties for air and ground and the boundary conditions, considering the air temperature in the buried pipe and the ground surface temperature as variables under cosinusoidal law. Moreover, the model allows us to evaluate the performance of a buried pipe of assigned length.

The model is obtained by the solution of heat and mass balances for air through the buried pipe, considering an appropriate temperature profile in the ground. This is determined by two methods: the first is based on Green's functions and the second, simplified, is based on the principle of superposition.

The agreement between the numerical and the experimental data has proved very satisfactory.

References

[1] M. Santamouris, G. Mihalakakou, D. Asimakoupolos, On the coupling of thermostatically controlled buildings with ground and night ventilation passive dissipation techniques, Solar Energy 60 (3–4) (1997) 191–197.
[2] J. Beck, K. Cole, A. Haji-Sheikh, B. Litkouhi, Heat Conduction Using Green's Functions, Hemisphere, New York, 1992.
[3] M. Cucumo, A. De Rosa, V. Ferraro, D. Kaliakatsos, Dimensionamento e prestazioni di condotti interrati ad aria, CDA 1 (2000).
[4] M. Cucumo, V. Ferraro, A. Vulcano, Un modello analitico monodimensionale per il dimensionamento degli scambiatori di calore interrati ad aria, in: XXIV Congresso Nazionale UIT sulla Trasmissione del Calore, Napoli, 2006.
[5] ASHRAE Fundamentals Handbook, Mass Transfer, 2001 (Chapter 5).
[6] E.R.G. Eckert, R.M. Drake, Heat and Mass Transfer, McGraw Hill Book Company, New York, 1959.
[7] P. Hollmuller, B. Lachal, Cooling and preheating with buried pipe systems: monitoring, simulation and economic aspects, Energy Build. 33 (2001) 509–518.
[8] A. Tzaferis, D. Liparakis, M. Santamouris, A. Argiriou, Analysis of the accuracy and sensitivity of eight models to predict the performance of earth-to-air heat exchangers, Energy Build. 18 (1) (1992) 35–43.
[9] S. Benkert, F.D. Heidt, D. Schöler, Calculation tool for earth heat exchanger GAEA, in: Proceeding of Building Simulation, vol. 2, Fifth International IBPSA Conference, Prague, 1997, pp. 9–16.
[10] M. De Paepe, A. Janssens, Thermo-hydraulic design of earth–air heat exchangers, Energy Build. 35 (2003) 389–397.
[11] J.W. Stevens, Coupled conduction and intermittent convective heat transfer from a buried pipe, Heat Transfer Eng. 23 (2002) 34–43.
[12] J.W. Stevens, Optimal placement depth of air–ground heat transfer systems, Appl. Therm. Eng. 24 (2004) 149–157.
[13] R. Wagner, S. Beisel, A. Spieler, K. Vajen, A. Gerber, Measurement, modeling and simulation of an earth to air heat exchanger in Marburg, Germany, in: Fourth ISES Europe Solar Congress, Kopenhagen, 2000.
[14] F. Sthål, Preheating of supply air trough an air tube system, energy demand and moisture consequence, in: Sixth Nordic Symposium on Building Physics, Trondheim, Norway, 17–19 June, 2002.
[15] G. Sharan, R. Jadhav, Performance of single pass earth-tube heat exchanger: an experimental study, J. Agric. Eng. (2003).
[16] V. Puri, Feasibility and performance curves for intermittent earth tube heat exchangers, Trans. ASAE 29 (2) (1986) 526–532.
[17] G. Mihalakakou, M. Santamouris, D. Asimakoupolos, Modelling the thermal performance of earth-to-air heat exchangers, Solar Energy 53 (3) (1994) 301–305.
[18] C.P. Jacovides, G. Mihalakakou, Underground pipes system as an energy source for cooling/heating purposes, Renew. Energy 6 (8) (1995) 893–900.
[19] J. Millette, N. Galanis, Yearly thermal analysis of a residential earth-tube heat exchanger, ASME J. Solar Energy 117 (1995) 22–29.
[20] C. Gauthier, M. Lacroix, H. Bernier, Numerical simulation of soil heat exchanger-storage systems for greenhouses, Solar Energy 60 (6) (1997) 333–346.
[21] J. Pfafferott, Evaluation of earth-to-air heat exchangers with a standardised method to calculate energy efficiency, Energy Build. 35 (2003) 971–983.
[22] A. Shukla, G.N. Tiwari, M.S. Sodha, Parametric and experimental study on thermal performance of an earth-air heat exchanger, Int. J. Energy Res. 30 (2006) 365–379.
[23] V.A.F. Costa, Thermodynamic analysis of building heating or cooling using the soil as heat reservoir, Int. J. Heat Mass Transfer 49 (2006) 4152–4160.
[24] M. Krarti, J.F. Kreider, Analytical model for heat transfer in an underground air tunnel, Energy Convers. Manage. 37 (10) (1996) 1561–1574.
[25] P. Hollmuller, Analytical characterization of amplitude-dampening and phase-shifting in air/soil heat exchangers, Int. J. Heat Mass Transfer 46 (2003) 4303–4317.
[26] H.S. Carlslaw, J.C. Jaeger, Conduction of Heat in Solids, second ed., Clarendon Press, Oxford, 1959.
[27] F.P. Incropera, D.P. De Witt, T.L. Bergman, A.L. Lavine, Fundamentals of Heat and Mass Transfer, sixth ed., Wiley, and Sons, 2006.
[28] Guglielmini, Pisoni, Elementi di Trasmissione del Calore, Ed. Veschi, Milano, 1990.
[29] E.A. Rodriguez, J.M. Cjudo, S. Alvarez, Earth-tube system performance, in: Proceeding CIB Meeting Air Quality and Air Conditioning, Paris, France, 1988.
[30] K. Boulama, N. Galanis, J. Orfi, Heat and mass transfer between gas and liquid streams in direct contact, Int. J. Heat Mass Transfer 47 (2004) 3669–3681.
[31] K.J. Albers, Untersuchungen zur Auslegung von Erdwärmeaustauschern für die Konditionierung der Zuluft für Wohngebäude, Ph.D Thesis, Universität Dortmund, Dortmund, 1991.